Gauge theory approach to branes and spontaneous symmetry breaking

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March 16, 2017

Abstract

Gauge theory approach to consideration of the Nambu-Goldstone bosons as gauge and vector fields represented by the Cartan forms of spontaneously broken symmetries, is discussed. The approach is generalized to describe the fundamental branes in terms of $(p + 1)$-dimensional worldvolume gauge and massless tensor fields consisting of the Nambu-Goldstone bosons associated with the spontaneously broken Poincare symmetry of the D-dimensional Minkowski space.

1 Introduction

Equivalence between explicit solution of the string equations and restoration of its worldsheet in the Minkowski space was observed by Regge and Lund [1] (see also [2-4]). This statement resulted from the differential geometry of embedded surfaces, in particular, from the first-order linear Gauss-Weingarten differential equations for the vectors tangent and normal to string worldsheet [5]. In the case of 4-dim. Minkowski space they found that the integrability condition for the G-W equations was just the sine-Gordon equation describing a surface embedded into 3-dimensional Euclidean space. This connection

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of the nonlinear string equations with the linear equations of the inverse scattering method [6] added a new sight to numerous efforts to overcome nonlinearities of the (mem)brane dynamics [7-19] .

The geometric approach by Regge-Lund was generalized to strings embedded into D-dim. Minkowskii space in [20], [21], where its reformulation in terms of the Yang-Mills theory was developed. The generalization was based on the Cartan method of moving frame [22] and its development by Volkov [23] under geometrization of the phenomenological Lagrangian method [24-27] . The gauge reformulation established equivalence between the string and a closed sector of states of the exactly integrable 2-dim. $SO(1,1) \times SO(D-2)$ invariant model of interacting gauge and massless scalar fields. This approach was recently generalized to the case of the fundamental Dirac p-branes embedded into D-dim. Minkowski space, where the p-brane turned out to be the exact solution of a $(p+1)$-dimensional model invariant under diffeomorphisms and $SO(D-p-1)$ gauge transformations [28]. The model contains the constrained $SO(D-p-1)$ multiplets of gauge and massless tensor fields. The latter represent the second fundamental form of $(p+1)$-dim. world hypervolume. This reformulation is based on the construction by Faddeev and Semenov-Tyan-Shansky showing the equivalence between chiral and constrained Yang-Mills field theories [29].

Here we develop this approach as an alternative way to describe the Nambu-Goldstone (N-G) fields of a spontaneously broken internal symmetry G in terms of the constrained vector and Yang-Mills multiplets of the unbroken subgroup $H \in G$. Such a description treats the multiplet components as new dynamical variables of an associated gauge invariant action. The gauge covariant constraints are the integrability conditions of the PDEs which express the N-G fields through the multiplet fields. These conditions are just the Maurer-Cartan (M-C) equations for the space G/H represented in terms of the gauge strength and vector fields. We explain how the constraints reduce the redundant components of the Cartan multiplets preserving the number of the physical DOF equal to the number of the essential N-G fields. Using these results we prove that the multiplets of the p-brane model [28] represent the N-G fields of the broken $ISO(1, D-1)$ global symmetry of $\mathbf{R}^{1,D-1}$. The Poincare symmetry breakdown is caused by the embedding of a $(p+1)$-dimensional hypersurface Σ_{p+1} into $\mathbf{R}^{1,D-1}$. We find that the brane vacuum manifold is fixed by the $(D-p-1)$ conditions: $\omega_a \equiv \mathbf{n}_a(\xi)d\mathbf{x}(\xi) = 0$ for the local translations orthogonal to Σ_{p+1} at each of its points $\mathbf{x}(\xi)$ [20]. These conditions are invariant under the *left global $ISO(1, D-1)$* and the

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right local $ISO(1,p) \times SO(D-p-1)$ transformations of the Cartan moving frame $\mathbf{n}_A(\xi)$ attached to Σ_{p+1} and its origin $\mathbf{x}(\xi)$. The world vectors $\mathbf{x}(\xi)$ and $\mathbf{n}_A(\xi)$ are treated as macroscopic translational and rotational order parameters, and Σ_{p+1} emerges as a world hypervolume swept by the Dirac p-brane. It follows from the $SO(D-p-1)$ gauge invariant action S_{Dir} (113) formulated in terms of the Cartan multiplets located on the minimal hypersurface Σ_{p+1}. This action is also invariant under diffeomorphisms because the gauge fields of the unbroken gauge subgroup $SO(1,p)$ are identified with the metric connection of Σ_{p+1}. The effective tensor multiplet $l_{\mu\nu}{}^a(\xi)$ encoding the broken rotational and translational N-G modes turns out to be the traceless second quadratic form of Σ_{p+1}. The modes associated with the translations in $\mathbf{R}^{1,D-1}$ are also presented in S_{Dir} by non-dynamical components of the background metric $g_{\mu\nu}(\xi)$ of Σ_{p+1}. These modes provide invariance of S_{Dir} under diffeomorphisms and generate the cosmological term $\sqrt{|g|}$ which is dynamical one in the standard Dirac-Nambu action. But in the gauge approach the dynamical equations for $g_{\mu\nu}$ are provided by the Gauss conditions. The latter, together with the Ricci embedding conditions, are taken into account in the Euler-Lagrange EOM following from S_{Dir}. For the case of string in 3-dim. Minkowski space the Gauss constraints turn out to be equivalent to the condition $R_{\mu\nu} = \kappa g_{\mu\nu}$ defining Einstein spaces [20]. We find that the G-C-R constraints considered as the initial data of the Cauchy problem for the brane EOM are conserved. It proves the uniqueness of the found solution of the Euler-Lagrange PDEs as a consequence of the Cauchy-Kowalevskaya theorem.

The nonlinear realization of the Poincare group broken to the discussed subgroup was used in many papers for description of generalized string and p-brane dynamics (see e.g. [30-36] and refs. there). Construction of effective string and brane actions including higher-derivative terms in the world vector $\mathbf{x}(\xi)$ was studied there. Various methods, including the use of the Cartan forms, were proposed to build the Lorentz and diffeomorphism invariant higher order terms. However, any systematic scheme producing such invariants faces the problem of their classification in each order in derivatives, because not all admissible invariants are independent. The Cartan form products together with their covariant exterior differentials make up a complete set of invariants contributing to the effective action. This observation was earlier taken into account in our paper [37], where a general method was proposed for constructing the individual terms of the expansion

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of the n-point dual amplitude with respect to homogeneous functions of degree $r = 1, 2, ...$ of the Mandelstam kinematic invariants s_{ik}. The fulfillment of the Adler's principle was ensured there by using a phenomenological Lagrangian invariant under the chiral symmetry and containing higher derivatives of the meson fields. The complete set of the invariants of the fourth order in the derivatives was built in [37] for an arbitrary semisimple symmetry group realized non-linearly. These results are used in the considered here gauge approach to branes. The Cartan multiplets and their covariant derivatives create covariant blocks for constructing higher order terms in effective action of strings and p-branes explicitly invariant under the Lorentz transformations and diffeomorphisms. This scheme is considered in Sections 2,3, where the general method of phenomenological Lagrangians is shortly dwelt on. Application of the discussed method to broken Poincare symmetry yields the desired higher order invariants. For example, the invariant terms in the gauge invariant actions for the Nambu-Goto string [20], [21] and Dirac p-brane [28] are quadratic in covariant derivatives of the second fundamental form and quartic in its components. In view of the generalized *Gauss Theorema Egregium* [28] the quartic terms are proportional to the ones quadratic in the Riemannian tensor components. Similar result was observed in [34], where for the first leading correction to the Nambu-Goto string action was found to be quadratic in the world sheet curvature. So, unification of the Cartan approach explicitly covariant under the Lorentz transformations and diffeomorphisms with the methods [34], [35] is promising and deserves a special investigation. It seems also interesting to apply the gauge approach to the quantization problem of Dirac branes. The point is that the gauge approach yields a map of the brane dynamics in the dynamics of a Yang-Mills model which obeys the universal embedding constraints for the world hypersurfaces swept by branes. The Cartan multiplets represent the covariant objects realizing the said map. The gauge invariant p-brane action S_{Dir} is suitable for application of the BRST-BFV method which has demonstrated its power in quantization of the Yang-Mills theories with constraints (see e.g. [38], [39]). This method will yield an important information about the ghost sector structure, anomalies and critical dimension in the theory of branes. The use of the gauge approach also gives a clear geometric explanation for many aspects of the classical brane dynamics reformulated in terms of the Cartan multipletd. So, the inverse Higgs phenomenon [40] which pulls the trigger for splitting N-G modes into essential and not essential ones means a simple choice for the first $(p + 1)$ vectors $\mathbf{n}_i(\xi)$ of the moving frame $\mathbf{n}_A(\xi)$ to

be tangent to Σ_{p+1}. It is equivalent to the above mentioned definition of the vacuum manifold by the conditions $\omega_a = 0$. Another example of the inverse Higgs conditions emerging in the gauge theory approach to branes is given by the minimality conditions $Spl^a \equiv g^{\mu\nu} l^a_{\mu\nu} = 0$ invariant under all symmetries of the action S_{Dir}.

2   Coset spaces and fiber bundles

Consider the Nambu-Goldstone fields generated by a global semisimple group of symmetry G with the algebra generators X_i and Y_α

$$[Y_\alpha, Y_\beta] = ic^\gamma{}_{\alpha\beta}Y_\gamma, \quad [X_i, Y_\alpha] = ic^k{}_{i\alpha}X_k, \quad [X_i, X_k] = ic^\alpha{}_{ik}Y_\alpha + ic^l{}_{ik}X_l. \quad (1)$$

If G is a spontaneously broken symmetry of a physical system with its vacuum invariant under the subgroup H, it is convenient to describe the corresponding N-G fields (in some neighborhood of the identity of G) with the help of the factorized representation of the group elements ([23],[26],[27])

$$G(a, b) = K(a)H(b), \tag{2}$$

where a and b parametrize the group space of G. The left multiplication of the global group G by any element $g \in G$

$$gG = G' \quad \rightarrow \quad gK(a)H(b) = K(a')H(b') \tag{3}$$

yields the following transformation rules of the parameters a and b

$$a' = a'(a, g), \quad b' = b'(b, a, g), \tag{4}$$

with the transformed parameters a' independent of b. This shows that the parameters a form the left coset space G/H invariant under non-linear transformations of G. Then these parameters may be mapped into the components of the N-G field $\pi(x)$ with the same nonlinear transformation law (4)

$$\pi'(x) = a'(\pi(x), g). \tag{5}$$

One can see that the left multiplication (3) preserves the form $G^{-1}dG$

$$G'^{-1}dG' = G^{-1}dG. \tag{6}$$

As a result, the expansion of the one-form (6) in the generators X_i, Y_α

$$G^{-1}dG = i\omega_G^i(a, b, da)X_i + i\theta_G^\alpha(a, b, da, db)Y_\alpha, \tag{7}$$

taking into account their algebra (1), creates the Cartan differential one-forms ω_G^i and θ_G^α [22] invariant under the left multiplications (3)

$$\omega_G^i(a', b', da') = \omega_G^i(a, b, da), \quad \theta_G^\alpha(a', b', da', db') = \theta_G^\alpha(a, b, da, db). \tag{8}$$

Therefore, it was proposed to use these forms as building blocks for construction of G-invariant phenomenological Lagrangians [23].

The parameters b in the forms (7) describe non-physical DOF and can be fixed like gauge parameters. This becomes clear after the substitution of expression (2) into $G^{-1}dG$ that yields the new representation for this form

$$G^{-1}dG = H^{-1}(K^{-1}dK)H + H^{-1}dH \tag{9}$$

which shows that the b-dependence of $G^{-1}dG$ arises as a result of H-subgroup gauge transformations of the b-independent form $K^{-1}dK$.

After taking into account Eq. (9), condition (6) transforms into

$$H'^{-1}(K'^{-1}dK')H' + H'^{-1}dH' = H^{-1}(K^{-1}dK)H + H^{-1}dH, \tag{10}$$

where $K'H' = gKH$, $K' \equiv K(a'(a, g))$, $H' \equiv H(b'(a, b, g))$.

Condition (10) may be rewritten in the form of the transformation law

$$K'^{-1}dK' = \tilde{H}(K^{-1}dK)\tilde{H}^{-1} + \tilde{H}d\tilde{H}^{-1}, \quad \tilde{H} := H'H^{-1} \tag{11}$$

that shows that the reduced Cartan form $K^{-1}dK$, depending only on the coset parameters a and their differentials

$$K^{-1}dK = i\omega^i(a, da)X_i + i\theta^\alpha(a, da)Y_\alpha, \tag{12}$$

transforms like the one-form of the gauge potential for H. The forms $\omega^i(a, da)$ and $\theta^\alpha(a, da)$ are the forms (8) calculated at the point a in the fixed coordinate frame with $b = 0$. Therefore, these forms are not invariant under the global G-shifts and their transformation rules

$$\omega'^i X_i = \omega^i(\tilde{H}X_i\tilde{H}^{-1}), \quad \theta'^\alpha Y_\alpha = \theta^\alpha(\tilde{H}Y_\alpha\tilde{H}^{-1}) - i\tilde{H}d\tilde{H}^{-1} \tag{13}$$

follow from Eq. (11) after using the commutation relations (1).

For infinitesimal transformations $g \in G$ in the fixed basis $b = 0$ one can write b' and $\tilde{H}$ in the following form

$$b' = \epsilon(a, 0, g), \qquad \tilde{H} \approx 1 + i\epsilon^\beta(a, 0, g)Y_\beta. \qquad (14)$$

As a result, Eqs. (13) are presented in the matrix form

$$\delta\omega^i X_i = i\epsilon^\beta[Y_\beta, X_i]\omega^i, \qquad \delta\theta^\alpha Y_\alpha = i\epsilon^\beta[Y_\beta, Y_\alpha]\theta^\alpha - d\epsilon^\gamma Y_\gamma. \qquad (15)$$

which realizes the adjoint representation of the vacuum subgroup H

$$\delta\omega^i = c^i{}_{k\beta}\epsilon^\beta\omega^k, \qquad \delta\theta^\alpha = -d\epsilon^\alpha + c^\alpha{}_{\gamma\beta}\epsilon^\beta\theta^\gamma. \qquad (16)$$

with the infinitesimal parameters ϵ^α depending on the coset coordinates a.

This shows that various combinations of the covariant Cartan forms ω^i (12) invariant under the gauge group H are automatically invariant under the left global transformations of G, and can be used for construction of the action of the N-G bosons [23]. The Cartan forms θ^α are the gauge potentials that permit to introduce covariant differentials for any multiplet of H. Thus, θ^α may be used not only for description of interactions between the N-G fields, but also of their interactions with other fields.

So, we have called to mind the well-known method of non-linear realizations using N-G fields associated with the spontaneously broken symmetry G. The method presents the parameter space of G as a fiber bundle where the coset G/H is the base and H is the fiber. A special accent has been made on some properties of the reduced Cartan one-forms ω^i, θ^α (12), because they realize linear representations, and can be alternatively used as new effective field variables instead of the usual coset coordinates a^λ. Due to the Gauss-Codazzi conditions for the Cartan forms and their gauge covariance, they do not produce new physical degrees of freedom, but only encode the N-G fields in terms of the constrained gauge multiplets of the vacuum subgroup H [29]. This approach to the description of N-G fields does not use any explicit parametrization of G and reveals their pure geometric roots based on the conception of connections and gauge fields. In this geometrical description of N-G fields their Euler-Lagrange equations, derived from a given phenomenological action, take the form of some gauge covariant constraints for the Cartan multiplets. To find the form of these constraints we need at least to choose an invariant phenomenological Lagrangian for the N-G fields. It will be considered in the next Section where some known examples of invariant Lagrangians formulated in terms of the Cartan forms will be given.

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3 Phenomenological Lagrangians

Going from the coset parameters a to the N-G fields $\pi(x)$ and using the definition of the differential one-forms ω^i [1]

$$\omega^i(\pi, d\pi) = \omega^i_\lambda(\pi)d\pi^\lambda \tag{17}$$

one can see that the Lame coefficients $\omega^i_\lambda(\pi)$ have the meaning of the vielbein components of the curved space G/H, that permits to write the squared element of length in the tangent space to G/H as

$$ds^2 = \omega_i\omega^i = \omega^i_\lambda\omega_{i\rho}d\pi^\lambda d\pi^\rho \equiv g_{\lambda\rho}(\pi)d\pi^\lambda d\pi^\rho, \tag{18}$$

where the invariant non-degenerate Cartan-Killing tensor built from the structure constants of the algebra (1) is used as a metric tensor to raise the subindex i of ω_i. The representation $g_{\lambda\rho}(\pi) = \omega^i_\lambda\omega_{i\rho}$ used in (18) is accompanied with the completeness condition

$$\omega^i_\lambda(\pi)\omega^\lambda_j(\pi) = \delta^i_j \tag{19}$$

that permits to invert relation (17) expressing $d\pi^\lambda$ in terms of $\omega^i(d)$

$$d\pi^\lambda = \omega^\lambda_i\omega^i(d), \tag{20}$$

and the space-time derivative $\partial_m\pi^\lambda \equiv \frac{\partial\pi^\lambda}{\partial x^m}$ through the vector field ω^i_m [2]

$$\partial_m\pi^\lambda = \omega^\lambda_i(\pi(x))\omega^i_m(\pi(x)), \quad \omega^i_m := \frac{\omega^i(d)}{dx^m}. \tag{21}$$

The relations show that the transition to the vector fields $\omega^i_m(\pi(x))$ from $\omega^i_\lambda(\pi(x))$ is realized by projecting them on the derivatives $\partial_m\pi^\lambda$, and this does not increase the DOF number. The simplest invariant action in the long-wave approximation for N-G fields expressed in terms of $\omega^i_m(\pi(x))$ is [23]

$$S = \frac{\gamma}{2}\int d^Dx\,\omega^i_m\omega^m_i \tag{22}$$

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[1]The greek letters $\lambda, \mu, ...$ from the second half of the alphabet numerate the coordinate indices of the curved space G/H.

[2]The latin letters $m, n, p, ...$ placed after the letter l in the alphabet are used for the space-time coordinates x^m.

and coincides with the action of massless scalar particles quadratic in $\partial_m \pi^\lambda(x)$ in the curved space G/H, but with its coordinates π^λ parametrized by x^m

$$S = \frac{\gamma}{2} \int d^D x \, g_{\lambda\rho}(\pi) \partial_m \pi^\lambda \partial^m \pi^\rho. \tag{23}$$

The corresponding EOM for the field $\pi^\lambda(x^m)$ is the geodesic equation

$$\partial_m \partial^m \pi^\rho + \Gamma^\rho_{\lambda\sigma} \partial_m \pi^\lambda \partial^m \pi^\sigma = 0 \tag{24}$$

in the Riemannian space G/H with its Christoffel symbols $\Gamma^\rho_{\lambda\sigma}(\pi^\sigma(x))$.

Another example of an invariant action is provided by using the gauge form θ^α for H that follows from (16). This form permits to construct the gauge covariant exterior differential $D \wedge \omega^i$ [3]

$$D \wedge \omega^i = d \wedge \omega^i + c^i{}_{k\alpha} \theta^\alpha \wedge \omega^k. \tag{25}$$

with a homogeneous gauge transformation under multiplications (3)

$$\delta(D \wedge \omega^i) = c^i{}_{k\beta} \epsilon^\beta (D \wedge \omega^k), \tag{26}$$

as it follows from the Jacobi identity produced by the cyclic sum of the commutators $[[Y_\alpha, Y_\beta], X_i] + \dots = 0$

$$c^l{}_{\beta k} c^k{}_{\alpha i} + c^l{}_{\alpha k} c^k{}_{i\beta} + c^l{}_{i\rho} c^\rho{}_{\beta\alpha} = 0. \tag{27}$$

As a result, we obtain the invariant action in four-dimensional space-time quartic in the derivatives of N-G fields

$$S = \frac{\tilde\gamma}{2} \int d^4 x \, D \wedge \omega_i \wedge D \wedge \omega^i. \tag{28}$$

However, this action turns out to be equal to the well-known action

$$S = \frac{\tilde\gamma}{8} c_{jk}{}^f c_{fil} \int d^4 x \, \omega^j \wedge \omega^k \wedge \omega^i \wedge \omega^l \tag{29}$$

proportional to the squared constant torsion of G/H, as it follows from the group structure Eq. (44) discussed in the next section [4] Thus, this action belongs to the above-discussed set of invariant actions polynomial in ω^i.

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[3]The symbols $\wedge$ and $d\wedge$ denote the exterior product and the external differential of the differential forms, respectively.

[4]The physical role of the affine connections corresponding to the constant torsions for the chiral groups $G \times G$ was cleared up in [41].

More general invariant actions in the D-dimensional Minkowski space can be built by using the covariant derivatives $D_n \omega_m^i$

$$D_n \omega_m^i := \partial_n \omega_m^i + c^i{}_{k\alpha} \theta_n^\alpha \omega_m^k \tag{30}$$

instead of the exterior differentials (25) connected with $D_n \omega_m^i$ by the relation

$$D \wedge \omega^i = D_\nu \omega_\mu^i d\pi^\nu \wedge d\pi^\mu. \tag{31}$$

Using the relation $\omega^i(d) = \omega_\mu^i d\pi^\mu = \omega_m^i dx^m$ one can rewrite (31) as

$$D \wedge \omega^i = D_n \omega_m^i dx^n \wedge dx^m. \tag{32}$$

The covariant derivatives $D_n \omega_m^i$ form linear representations of the gauge group H and the Lorentz group in D-dimensional Minkowski space. So, invariant combinations of $D_n \omega_m^i$, such as $D_n \omega_i^n D_m \omega^{mi}$ or $D_n \omega^{mi} D^n \omega_{mi}$ as well as their higher monomials, are invariants of the left shifts of G. These combinations form a subset of building blocks composing a general invariant phenomenological Lagrangian.

At last, we have the covariant two-form F^α of the gauge strength for θ^γ

$$F^\alpha := D \wedge \theta^\alpha = d \wedge \theta^\alpha + \frac{1}{2} c^\alpha{}_{\gamma\beta} \theta^\beta \wedge \theta^\gamma. \tag{33}$$

It also undergoes homogeneous gauge transformations under H induced by the left shifts (3)

$$\delta F^\alpha = c^\alpha{}_{\gamma\beta} \epsilon^\beta F^\gamma, \tag{34}$$

as it follows from the Jacoby identity for the structure constants of H

$$c^\gamma{}_{\alpha\bullet} c^\bullet{}_{\nu\rho} + c^\gamma{}_{\nu\bullet} c^\bullet{}_{\rho\alpha} + c^\gamma{}_{\rho\bullet} c^\bullet{}_{\alpha\nu} = 0.$$

For a semisimple Lie algebra one can choose a basis in which the structure constants with all lower or upper indices are completely antisymmetric. As a result, the combinations of the 2-form gauge strength F^γ invariant under the subgroup H are invariant under nonlinear transformations of the group G, and may also be used for construction of G-invariant actions including higher order terms in the derivatives of π^λ. The kinetic term for the gauge field $\theta_m^\alpha(\pi)$ composed from the N-G fields

$$S = -\frac{\tilde{\gamma}}{4} \int d^D x \, F_{mn}^\alpha F_\alpha^{mn} \tag{35}$$

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is an example of the Lagrangian of the fourth order in the derivatives of π^λ invariant under the group G. There are other important invariants.

So, the Cartan forms make up a complete set of covariant blocks for construction of general phenomenological action. However, not all monomials constructed from the Cartan forms are linearly independent. he Cartan structure equations play an important role in search for the number of independent invariants.

This short survey of the well-known description of N-G fields allows to consider their above-mentioned description in terms of new effective dynamical variables: the fields ω_m^i, θ_m^α forming the linear Cartan multiplets.

4   N-G fields as Cartan multiplets

A transition from some dynamical variables to other ones implies the choice of transition functions connecting these two sets. The transition has to preserve the number of the original physical degrees of freedom. When the number of new variables is larger than the number of the original variables, then the corresponding constraints and/or a gauge symmetry have to be added to reduce the abundance. The latter case is realized by the transition from the N-G fields $\pi^\lambda(x)$ to the massless composite vector $\omega_m^i(\pi(x))$ and the gauge fields $\theta_m^\alpha(\pi(x))$ forming the Cartan multiplets. Indeed, the definition (12) of the ω^i and θ^α differential forms can be considered to be the matrix system of PDEs

$$dK = i\omega^i K X_i + i\theta^\alpha K Y_\alpha, \tag{36}$$

for the matrix $K(\pi(x))$ as a function of the given coefficients $\omega_m^i(\pi(x))$ and $\theta_m^\alpha(\pi(x))$. Eqs. (36) express the total differential dK through the products of K with the Cartan forms. As a result, the N-G fields are presented as implicit functions of the massless multiplets. Solving PDEs (36) one can restore the original N-G fields encoded by the Cartan multiplets. The PDEs are rather nontrivial and put severe constraints on the Cartan multiplets. The constraints follow from the integrability conditions of Eqs. (36)

$$d \wedge (\omega^i(\pi, d\pi)K) X_i + d \wedge (\theta^\alpha(\pi, d\pi)K) Y_\alpha = 0 \tag{37}$$

which transform into the Cartan group structure equations after using (1)

$$d \wedge \omega^k = \frac{1}{2} c^k{}_{ij} \omega^i \wedge \omega^j - c^k{}_{i\beta} \theta^\beta \wedge \omega^i, \tag{38}$$

$$d \wedge \theta^\alpha = \frac{1}{2} c^\alpha{}_{ij} \omega^i \wedge \omega^j - \frac{1}{2} c^\alpha{}_{\gamma\beta} \theta^\beta \wedge \theta^\gamma. \tag{39}$$

To see that the new variables presented by the Cartan multiplets do not contain any abundant *physical* DOF, we note that Eqs. (38-39) are equivalent to the Maurer-Cartan (M-C) equations defining the geometric characteristics of the space G/H [22], [23]. By solving these equations one can restore the N-G fields identified with the coordinates of G/H modulo its motion as a whole and coordinate reparametrizations. The mentioned M-C equations

$$d \wedge \omega^i + \omega^i{}_k \wedge \omega^k = \frac{1}{2} c^i{}_{jk} \omega^j \wedge \omega^k, \tag{40}$$

$$d \wedge \omega^k{}_l + \omega^k{}_j \wedge \omega^j{}_l = \frac{1}{2} c^k{}_{l\beta} c^\beta{}_{ij} \omega^i \wedge \omega^j. \tag{41}$$

follow from (38-39) after using the definition of the spin connection $\omega^i{}_k(d)$

$$\omega^i{}_k(d) = c^i{}_{k\beta} \theta^\beta(d) \tag{42}$$

and describe the spaces with constant torsion and curvature tensors

$$T^i{}_{jk} = c^i{}_{jk}, \qquad R^k{}_{lij} = c^k{}_{l\beta} c^\beta{}_{ij}. \tag{43}$$

To obtain Eqs. (40-41) there were used the Jacobi identities (27). e The l.h.s. of Eqs. (40-41) coincide with the covariant differentials (25) and (33), where $\theta^\alpha(d)$ is changed by the spin connection one-form $\omega^i{}_k(d)$. This substitution creates the covariant differential $D \equiv D^{spin}$ in the curved space G/H and presents its M-C Eqs. (40-41) in the equivalent form

$$D \wedge \omega^i = \frac{1}{2} T^i{}_{jk} \omega^j \wedge \omega^k, \tag{44}$$

$$D \wedge \omega^i{}_k = \frac{1}{2} R^i{}_{kjl} \omega^j \wedge \omega^l. \tag{45}$$

Eqs. (45) show that the *physical* DOF represented by the two-form gauge strength F^α (33) are covariantly expressed through the constant Riemannian tensor of G/H projected on the one-forms ω^i of the vector Cartan multiplet

$$F^i{}_j \equiv c^i{}_{j\alpha} F^\alpha = R^i{}_{jkl} \omega^k \wedge \omega^l. \tag{46}$$

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For the torsionless Riemannian spaces with $T^i{}_{jk} = 0$ their spin connection $\omega_\nu{}^i{}_k$ turns out to be expressed through ω_i by the relation [28]

$$\Gamma^\rho_{\nu\lambda} = \omega^\rho_i \omega_\nu{}^i{}_k \omega^k_\lambda + \partial_\nu \omega^k_\lambda \omega^\rho_k \equiv \omega^\rho_i D^{spin}_\nu \omega^i_\lambda, \tag{47}$$

where $\partial_\mu \equiv \frac{\partial}{\partial \pi^\mu}$, and D^{spin}_ν is the above defined covariant derivative D_ν (30)

$$D_\mu \omega^i_\nu \equiv D^{spin}_\mu \omega^i_\nu = \partial_\mu \omega^i_\nu + \omega_\mu{}^i{}_k \omega_\nu{}^k. \tag{48}$$

In their turn the DOF represented by ω^j are restricted by constraints (44) and the Lagrangian EOM for the N-G fields $\pi^\lambda(x)$ considered below.

The change-over from the covariant exterior differentials to the covariant derivatives in Eqs. (44-45) transforms the latter into the system of the first-order PDEs for the Cartan multiplets ω^i_m and θ^α_m

$$D_{[m} \omega^i_{n]} = T^i{}_{jk} \omega^j_m \omega^k_n, \tag{49}$$

$$F_{mn}{}^i{}_j = R^i{}_{jkl} \omega^k_m \omega^l_n, \tag{50}$$

where $[.m.n]$ means antisymmetrization in the space-time indices m, n.

So, constraints (49-50) provide the balance between the *physical* DOF represented by the Cartan multiplets and by the coset fields π^λ.

Now we discuss the representation of the EOM discussed in Section 3 in terms of the Cartan multiplets. As an example, we consider action (22) and after taking into account (47) find the expression of geodesic Eqs. (24) in terms of the Cartan multiplets. The substitution of $\Gamma^\rho_{\nu\lambda}$ (47) into (24) yields

$$\partial^m \pi^\rho_{,m} + \omega^\rho_i \pi^{\lambda,m} (D_\lambda \omega^i_\sigma) \pi^\sigma_{,m} = 0,$$

where $\pi^\rho_{,m} \equiv \partial_m \pi^\rho$, and we obtain

$$\partial^m \pi^\rho_{,m} + \omega^\rho_i \pi^{\lambda,m} D_\lambda \omega^i_m - \pi^{\lambda,m} \partial_\lambda \pi^\rho_{,m} = 0$$

after taking into account relations (19), (21). The first and the third terms in the preceeding equation are mutually cancelled in view of the relation $D_m \omega^{im} = \pi^\lambda_{,m} D_\lambda \omega^{im}$, and EOM (24) transform into the gauge covariant condition for the massless vector multiplet ω^i_m

$$D_m \omega^{im} = 0. \tag{51}$$

Eq. (51) gives an example of the additional constraints implied by the EOM generated by the action (23) quadratic in $\partial_m \pi^\lambda$. Thus, the N-G bosons π^λ,

parametrizing the *symmetric* space G/H and described by S (23), can be equivalently described by the massless vector multiplet ω_m^i interacting with the Yang-Mills field θ_m^α provided the constraints (49-50), (51)

$$D_{[m}\omega_{n]}^i = 0,$$
$$F_{mn}{}^i{}_j = c^i{}_{j\beta}c^\beta{}_{kl}\omega_m^k\omega_n^l, \qquad (52)$$
$$D^m\omega_m^i = 0.$$

are satisfied. The conclusion can be extended to more general Lagrangians including the higher order invariants, because constraints (49-50) do not depend on the Lagrangian choice. On the contrary, EOM depend on the Lagrangian and, therefore yield new constraints instead of (51) together with a new gauge invariant action expressed in terms of the Cartan multiplets.

The above statement accompanied with the gauge invariant action was observed by Faddeev and Semenov-Tyan-Shanskii in [29], where they started from the *invariant* Cartan forms (7). However, instead of fixing the coordinate frame by the above-used condition $b = 0$, explicitly removing non-physical N-G fields, they extended the global left symmetry (3) of the phenomenological Lagrangian by its *gauge* symmetry under the *right* multipications $G' = Gh$ with $h \in H$. In view of this gauge invariance the redundant N-G fields associated with the parameters of H turned out to be non-physical DOF removed by gauge fixing. The use of G-invariant Cartan forms (7) as building blocks in effective gauge Lagrangians ensures their invariance under the global left G-multiplications. But they have also to be invariant under the right gauge symmetry to preserve the number of the original DOF.

The transformation rules of ω_G^i, θ_G^α (7) under the right gauge shifts

$$G' = Gh \quad \rightarrow \quad G'^{-1}dG' = h^{-1}(G^{-1}dG)h + h^{-1}dh \qquad (53)$$

have the form of the following gauge transformations

$$\omega_G'^i X_i = \omega_G^i h^{-1} X_i h, \quad \theta_G'^\alpha Y_\alpha = \theta_G^\alpha h^{-1}Y_\alpha h - ih^{-1}dh. \qquad (54)$$

The use of (1) makes it possible to rewrite relations (54) in the form

$$\delta\omega_G^k = -c^k{}_{i\beta}\epsilon^\beta\omega_G^i, \qquad \delta\theta_G^\gamma = d\epsilon^\gamma - c^\gamma{}_{\alpha\beta}\epsilon^\beta\theta_G^\alpha \qquad (55)$$

of the infinitesimal transformations $h \approx 1 + i\epsilon^\beta Y_\beta$ with the space-time dependent parameters ϵ^β differing from (16) by the change of the sign $\epsilon^\beta \rightarrow -\epsilon^\beta$.

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This proves equivalence between the standard description of N-G bosons [24-27], where they are realized as coordinates of some G/H, and their description as the Cartan multiplets of the right gauge group H. Using this statement we will show that p-brane action [28] in D-dimensional Minkowski space is interpreted in terms of fields for the spontaneously broken Poincare symmetry $ISO(D-1)$ with the coset $ISO(1, D-1)/ISO(1, p) \times SO(D-p-1)$. Thereat, the Cartan formalism of moving frames discussed below works as a key mathematical implement.

5   Moving frame in Minkowski space

The moving frame in the Minkowski space $\mathbf{R}^{1,D-1}$ with the global coordinates $\mathbf{x} = \{x^m\}, (m = 0, 1, ..., D-1)$ is formed by the orthonormal vectors $\mathbf{n}_A(\mathbf{x})$

$$\mathbf{n}_A(\mathbf{x})\mathbf{n}_B(\mathbf{x}) = \eta_{AB}, \quad (A, B = 0, 1, .., D-1), \tag{56}$$
$$d\mathbf{x} = \omega^A(d)\mathbf{n}_A, \quad d\mathbf{n}_A = -\omega_A{}^B(d)\mathbf{n}_B$$

with their vertex at the point $\mathbf{x}$. In mathematics a frame is defined as the pair $(\mathbf{x}, \mathbf{n}_A(\mathbf{x}))$ called the moving D-hedron. The Cartan differential forms $\omega^A(d)$ and $\omega_A{}^B(d)$ emerge in the form of infinitesimal *translational* and *rotational* componets of the D-hedron, respectively. In view of the $\frac{1}{2}D(D+1)$ constraints (56) one can interprete the D^2 dependent componets $n_{mA}(\mathbf{x})$ of the vectors $\mathbf{n}_A$ as elements of a pseudoorthogonal matrix $\hat{n} = n_{mA}$ parametrized by $\frac{1}{2}D(D-1)$ independent parameters π^λ. The latter are identified with the parameters of the $SO(1, D-1)$ group. Then the corresponding fields $\pi^\lambda(\mathbf{x})$ can be treated as the N-G fields of the completely broken $SO(1, D-1)$ symmetry. So, the frame $\mathbf{n}_A(\mathbf{x})$ gives an *implicit* representation of the N-G fields of the maximally broken Lorentz symmetry when $H = I$. Non-linear transformations of π^λ under the *left* multiplications from the global Lorentz group $SO(1, D-1)$ follow from the matrix multiplication

$$n'_{mA} = l_m{}^k n_{kA}, \tag{57}$$

where $\hat{l}$ is a Lorentz transformation matrix $l_m{}^p l^n{}_p = \delta_m{}^n$. Having found the transformation law for $\pi^\lambda(\mathbf{x})$ from Eqs. (57) one can build an invariant Lagrangian for the completely broken $SO(1, D-1)$ symmetry.

On the other hand, frame treated as a matrix carries a *right* index A, and one can consider the *right* action of a new *gauge* group $SO_R(1, D-1)$

$$\mathbf{n}'_A = -L_A{}^B(\pi^\lambda)\mathbf{n}_B, \quad L_A{}^B L^C{}_B = \delta_A^C \tag{58}$$

which preserves the constraints (56). As a result, the matrix $\hat{n}(\pi^\lambda)$ is covariant under the *left* global and *right local* transformations

$$n'_{mA} = -l_m{}^k n_{kB} (L^{-1})^B{}_A \tag{59}$$

forming the group $SO(1, D-1) \times SO_R(1, D-1)$ which preserves (56).

But, involvement of $SO_R(1, D-1)$ as a new gauge symmetry for a Lagrangain corresponding to the complete symmetry breaking will allow to remove all N-G fields. However, the situation changes when the right gauge symmetry is chosen to be a *subgroup* H_R of $SO_R(1, D-1)$, because it allows to remove only the N-G fields corresponding to the generators of the *unbroken* symmetry H. Nevertheless, the discussed example of the complete breaking of the global $SO(1, D-1)$ symmetry is instructive for illustration of the connection between the frame and the *left* invariant Cartan form $\hat{n}^{-1} d\hat{n}$

$$\omega_A{}^B(d) = (\hat{n}^{-1} d\hat{n})_A{}^B \equiv \mathbf{n}_A d\mathbf{n}^B. \tag{60}$$

An infinitesimal *right* local rotation from $SO_R(1, D-1)$ applied to $\mathbf{n}_A$

$$\delta \mathbf{n}_A = \mathbf{n}_B \epsilon^B{}_A \tag{61}$$

results in the following gauge transformation of $\omega_A{}^B(d)$

$$\delta \omega_A{}^B(d) = d\epsilon_A{}^B + [\omega(d), \epsilon]_A{}^B \tag{62}$$

with the commutator $[\hat{\omega}, \hat{\epsilon}]$ in the r.h.s.. We see, that the left invariant $\hat{\omega}$-form transforms as a gauge one-form under the right transformations from $SO_R(1, D-1)$. This demands the use of the covariant differential $D_A{}^B$

$$D_A{}^B = \delta_A{}^B d + \omega_A{}^B(d). \tag{63}$$

for a vector field $\mathbf{V} = V^A \mathbf{n}_A$ transforming as $\delta V_A = V_B \epsilon^B{}_A$ that results in

$$\delta(DV)_A = (DV)_B \epsilon^B{}_A. \tag{64}$$

Using the covariant differential (63) we present (62) in the standard form

$$\delta\hat{\omega} = [D, \hat{\epsilon}]. \tag{65}$$

and the gauge covariant 2-form of the strength $F_A{}^B$ built from $\omega_A{}^B(d)$

$$F_A{}^B := D_A{}^C \wedge D_C{}^B = (d \wedge \omega + \omega \wedge \omega)_A{}^B \tag{66}$$

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using the exterior product of the covariant differentials.

In Secton 4 it has been explained that the description of N-G fields as gauge fields needs to satisfy proper integrability conditions. In the discussed case these conditions are the integrability ones for Eqs.(60) which can be rewritten as PDEs for the frame components

$$d\mathbf{n}_A = -\omega_A{}^B(d)\mathbf{n}_B. \tag{67}$$

Then the integrability conditions for Eqs.(67) are presented in the form

$$d \wedge \omega_A{}^B + \omega_A{}^C \wedge \omega_C{}^B = 0 \quad \rightarrow \quad F_A{}^B = 0. \tag{68}$$

Eqs. (68) show that the one-form $\omega_A{}^B$ treated - as a gauge potential for the *maximal right* gauge symmetry $SO_R(1, D-1)$ - represents the pure gauge DOF. This agrees with the above made statement that the addition of $SO_R(1, D-1)$ as a new Lagrange gauge symmetry for the case of the *complete* breakdown of the Lorentz group removes all rotational N-G fields.

However, in the case of p-brane the Lorentz symmetry is *partially* broken to its subgroup $H = SO(1,p) \times SO(D-p-1)$. Then the *right* gauge group $SO_R(1, D-1)$ is changed by its subgroup $H_R = SO_R(1,p) \times SO_R(D-p-1)$. Thereat, there will be excluded only the N-G fields corresponding to the unbroken subgroup H. The remaining N-G bosons will be presented by the Cartan multiplets described by the invariant action [28].

6 Hypersurfaces and moving frames

Consider a $(p+1)$-dim. world hypersurface Σ_{p+1} embedded into the space $\mathbf{R}^{1,D-1}$ invariant under the global Poincare group $ISO(1, D-1)$. The world coordinates of $\mathbf{x}(\xi^\mu)$ of Σ_{p+1} depend on its $(p+1)$ internal coordinates $\xi^\mu = (\tau, \sigma^r)$, $(r = 1, 2, .., p)$. One can assume that Σ_{p+1} is hyper-volume swept by a p-brane. The fields $\mathbf{n}_A(\xi)$ (56) attached to Σ_{p+1} can fix its orientation at each point $\mathbf{x}(\xi^\mu)$. Any fixation of the origin and orientation of $\mathbf{n}_A$ means the choice of a vacuum state of a p-brane. Using arbitrariness in orientation of $\mathbf{n}_A$ one can fix the manifold of degenerated vacuum states by the condition for the first $p+1$ vectors $\mathbf{n}_i$, $(i, k = 0, 1, ..., p)$ to be tangent to Σ_{p+1} at each point $\mathbf{x}(\xi^\mu)$. The residual rotational symmetry of so chosen vacuum manifold is described by the $SO(1,p) \times SO(D-p-1)$ of the local right rotations of $\mathbf{n}_A(\xi)$. Requirement of this symmetry is equivalent to spontaneous breaking

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of the left global symmetry $SO(1, D-1)$ to $SO(1,p) \times SO(D-p-1)$ in correspondence with our previous discussion. Then the rotational N-G fields can be described by the above-discussed Cartan multiplets. A fixing of the order parameter $\mathbf{x}(\xi)$ of Σ_{p+1} will spontaneously break the global translational symmetry of $\mathbf{R}^{1,D-1}$. The subgroup $SO(1,p)$ consists of the Lorentz rotations acting in the planes tangent to Σ_{p+1} which are added by independent $SO(D-p-1)$ rotations acting in $(D-p-1)$-dim. subspaces normal to Σ_{p+1}. The subspaces are spanned by the orts $\mathbf{n}_a$ $(a, b = p+1, p+2, ..., D-p-1)$. Thus, our choice of the vacuum manifold splits the frame vectors $\mathbf{n}_A$ (56), originally encoding the $\frac{1}{2}D(D-1)$ N-G bosons of the broken Lorentz group, into two subsets: $\mathbf{n}_A = (\mathbf{n}_i, \mathbf{n}_a)$. These subsets form linear representations of the right local subgroups $SO(1,p)$ and $SO(D-p-1)$, respectively. As a result, among the $\frac{1}{2}D(D-1)$ rotational N-G bosons there are $\frac{1}{2}(p+1)p + \frac{1}{2}(D-p-1)(D-p-2)$ representing non-physical DOF which can be removed by gauge fixing.

The remaining $(p+1)(D-p-1)$ DOF correspond to the real rotational N-G bosons identified with the coordinates of the coset $SO(1, D-1)/SO(1,p) \times SO(D-p-1)$, and can be described in terms of the constrained Cartan vector and gauge multiplets. These multiplets form the blocks of the differential matrix one-form $\omega_A{}^B$ (60) belonging to the Lorentz algebra $SO(1, D-1)$

$$\omega_A{}^B(d) = \begin{pmatrix} A_i{}^k & W_i{}^b \\ W_a{}^k & B_a{}^b \end{pmatrix}. \tag{69}$$

The diagonal one-form submatrices $A_{\mu i}{}^k d\xi^\mu$ and $B_{\mu a}{}^b d\xi^\mu$ in (69) describe the gauge fields in the fundamental representations of the unbroken $SO(1,p)$ and $SO(D-p-1)$ subgroups, respectively. The off-diagonal submatrix $W_{\mu i}{}^b d\xi^\mu$, having $(p+1)(D-p-1)$ components equal to the number of N-G bosons, describes a charged vector multiplet in the bi-fundamental representation of the subgroup $SO(1,p) \times SO(D-p-1)$ with the covariant derivative

$$(D_\mu W_\nu)_i{}^a = \partial_\mu W_{\nu i}{}^a + A_{\mu i}{}^k W_{\nu k}{}^a + B_\mu{}^a{}_b W_{\nu i}{}^b. \tag{70}$$

The covariant derivatives $D_\mu^{\parallel}$ and $D_\mu^{\perp}$ associated with the gauge fields $\hat{A}_\mu$ and $\hat{B}_\mu$, respectively, are given by

$$D_\mu^{\parallel} \Phi^i = \partial_\mu \Phi^i + A_\mu{}^i{}_k \Phi^k, \tag{71}$$
$$D_\mu^{\perp} \Psi^a = \partial_\mu \Psi^a + B_\mu{}^a{}_b \Psi^b. \tag{72}$$

In terms of these covariant derivatives the strengths $F_{\mu\nu i}{}^k$, $H_{\mu\nu a}{}^b$ for $\hat{A}_\mu$ and $\hat{B}_\mu$, respectively, are presented as follows

$$F_{\mu\nu i}{}^k \equiv [D_\mu^{||}, D_\nu^{||}]_i{}^k = (\partial_{[\mu}A_{\nu]} + A_{[\mu}A_{\nu]})_i{}^k \tag{73}$$

$$H_{\mu\nu a}{}^b \equiv [D_\mu^\perp, D_\nu^\perp]_a{}^b = (\partial_{[\mu}B_{\nu]} + B_{[\mu}B_{\nu]})_a{}^b. \tag{74}$$

Then the integrability conditions (68) take the form of the constraints

$$F_{\mu\nu i}{}^k = -(W_{[\mu}W_{\nu]})_i{}^k, \tag{75}$$

$$H_{\mu\nu a}{}^b = -(W_{[\mu}W_{\nu]})_a{}^b, \tag{76}$$

$$(D_{[\mu}W_{\nu]})_i{}^a = 0 \tag{77}$$

which are the Gauss-Ricci-Codazzi eqs. reformulated in terms of the massless vector multiplet $W_{\mu i}{}^a$ and the gauge strenghts $\hat{F}_{\mu\nu}$, $\hat{H}_{\mu\nu}$ of the unbroken subgroup of the gauge Lorentz group attached to the base hypersurface [28].

The geometry of Σ_{p+1} is described by the induced pseudo-Riemannian metric $g_{\mu\nu}(\xi)$ defining the Levi-Chivita connection $\Gamma^\rho_{\mu\nu}$

$$g_{\mu\nu}(\xi) = \partial_\mu\mathbf{x}\partial_\nu\mathbf{x}, \quad \Gamma^\rho_{\mu\nu} = \frac{1}{2}g^{\rho\gamma}(\partial_\mu g_{\nu\gamma} + \partial_\nu g_{\mu\gamma} - \partial_\gamma g_{\mu\nu}) \tag{78}$$

which must be added into the gauge-covariant derivative $(D_\mu W_\nu)_i{}^a$ (70)

$$D_\mu W_{\nu i}{}^a \rightarrow \hat{\nabla}_\mu W_{\nu i}^a = \partial_\mu W_{\nu i}{}^a + A_{\mu i}{}^k W_{\nu k}{}^a + B_\mu{}^a{}_b W_{\nu i}{}^b - \Gamma^\rho_{\mu\nu}W^a_{\rho i} \tag{79}$$

that makes it covariant under reparametrizations of Σ_{p+1}. In view of the symmetry $\Gamma^\rho_{\mu\nu} = \Gamma^\rho_{\nu\mu}$ the covariantization changes not the G-R-C constraints (75-77), but the commutator of the derivatives (70)

$$[\hat{\nabla}_\mu, \hat{\nabla}_\nu] = \hat{F}_{\mu\nu} + \hat{H}_{\mu\nu} + \hat{R}_{\mu\nu} \tag{80}$$

by adding the Riemann-Cristoffel tensor of the metric $g_{\mu\nu}$ (78)

$$R_{\mu\nu}{}^\gamma{}_\lambda = \partial_{[\mu}\Gamma^\gamma_{\nu]\lambda} + \Gamma^\gamma_{[\mu|\rho}\Gamma^\rho_{|\nu]\lambda}. \tag{81}$$

The invariant Cartan forms for the generators of *global* translations of $\mathbf{R}^{1,D-1}$ are equal to dx^m, where x^m are the global Cartesian coordinates in Minkowski space. The projections of $d\mathbf{x}$ on the local frame vectors $\mathbf{n}_A(\xi)$ coincide with components of the local translational Cartan form ω^A (56)

$$\omega^A(d) = d\mathbf{x}\mathbf{n}^A(\xi) \equiv dx^m n_m{}^A \tag{82}$$

which is the D-bein one-form of the Minkowski space referred to the local frame. Eqs.(82) show the role of $n_m^A(x) = \frac{\omega^A(d)}{dx^m}$ as a local Lorentz matrix $n_m{}^A n_{nA} = \eta_{mn}$ connecting the components of the global and local vielbein one-forms. So, we can treat (82) as the PDEs which express the local translations $\omega^A(d)$ through $d\mathbf{x}$. The integrability conditions for Eqs. (82)

$$d \wedge \omega_A + \omega_A{}^B \wedge \omega_B = 0 \tag{83}$$

reproduce the second set of the M-C equations for the Minkowski space additional to the first set (68). Eqs. (68) and (83) form the M-C structure equations of the Minkowski space which is the homogeneous space $ISO(1, D-1)/SO(1, D-1)$ for the global Poincare symmetry.

After localization of the translation parameters $\epsilon^m(\xi^\mu)$ on Σ_{p+1} the transformation law for $dx^m(\xi)$ changes into the form

$$\delta dx^m = d\epsilon^m \tag{84}$$

like an abelian gauge one-form. The splitting $\mathbf{n}_A = (\mathbf{n}_i, \mathbf{n}_a)$ induces the splitting $\omega_A(\xi) = (\omega_i, \omega_a)$ into the tangent and orthogonal components to Σ_{p+1} at each of its points. In the chosen vacuum manifold $\mathbf{n}_i$ are tangent to Σ_{p+1} that is equivalent to the orthogonality conditions

$$\omega_a \equiv \mathbf{n}_a(\xi) d\mathbf{x}(\xi) = 0 \quad \rightarrow \quad d\mathbf{x} = \omega^i(d)\mathbf{n}_i(\xi). \tag{85}$$

In view of invariance of $\omega_a(d)$ under the left $ISO(1, D-1)$ global transformations one can, of course, interpret the conditions $\omega_a = 0$ similarly to the inverse Higgs phenomenon [40]. In our case, however, these conditions mean the choice of $ISO(1, p) \times SO(D-p-1)$ as a vacuum subgroup for the Poincare symmetry expressing physical equivalence of the remaining orientations and positions of Σ_{p+1} in the Minkowski space. Invariance of (85) under the *right local* $ISO(1, p) \times SO(D-p-1)$ transformations of the moving polyhedron $(\mathbf{x}(\xi), \mathbf{n}_A(\xi))$ shows that the Poincare symmetry is spontaneously broken.

In view of $\omega_a = 0$ the quadratic element $ds^2 = d\mathbf{x}^2$ of Σ_{p+1} takes the form

$$ds^2 = \omega_i \omega^i = \omega_\mu^i \omega_{i\nu} d\xi^\mu d\xi^\nu \equiv g_{\mu\nu}(\xi) d\xi^\mu d\xi^\nu. \tag{86}$$

This shows that the values $\omega_\lambda^i(\xi)$ are the components of a $(p+1)$-bein of Σ_{p+1} in terms of which its induced metric $g_{\mu\nu}(\xi)$ is

$$g_{\mu\nu} := \mathbf{e}_\mu \mathbf{e}_\nu = \omega_\mu^i \eta_{ik} \omega_\nu^k, \tag{87}$$

where $\mathbf{e}_\mu(\xi)$ is the natural frame on Σ_{p+1}

$$\mathbf{e}_\mu = \omega_\mu^i \mathbf{n}_i, \quad \omega_\mu^i \omega_k^\mu = \delta_k^i. \tag{88}$$

So, the unbroken translational modes from $\mathbf{x}(\xi)$ are condensed in the $(p+1)$-bein ω_λ^i and the metric $g_{\mu\nu}$. The dynamical role of the broken translational modes becomes clear after the substitution of $(D - p - 1)$ conditions $\omega^a = 0$ into the integrability conditions (83) that results in their splitting

$$D_{[\mu}^{||}\omega_{\nu]}^i \equiv \partial_{[\mu}\omega_{\nu]}^i + A_{[\mu}{}^i{}_k\omega_{\nu]}{}^k = 0, \tag{89}$$

$$\omega_{[\mu}^i W_{\nu]ia} = 0 \tag{90}$$

connecting ω_μ^i with the fields $W_{\mu i}{}^a$ and $A_\mu{}^i{}_k$ (70).
    The solution of constraints (90)

$$W_{\mu i}{}^a = -l_{\mu\nu}{}^a \omega_i^\nu, \tag{91}$$

reveals that the rotational N-G modes $\hat{W}_\mu$ referred to the natural frame $\mathbf{e}_\mu(\xi)$ (88) turn out to be components of the second fundamental form $l_{\mu\nu}{}^a$ of Σ_{p+1}

$$l_{\mu\nu}{}^a := \mathbf{n}^a \partial_{\mu\nu}\mathbf{x}. \tag{92}$$

Thus, we find that the fixation of the vacuum manifold by the conditions $\omega^a = 0$ makes the N-G bosons of the broken translations shifted into the second fundamental form. This permits to express the N-G bosons of the broken Lorentz transformations and translations through the components of $l_{\mu\nu}^a$.
    Another effect of the vacuum conditions follows from Eqs. (89) which are equivalent to the conditions of the parallel transport of ω_μ^i along Σ_{p+1}

$$\nabla_\mu^{||}\omega_\nu^i \equiv \partial_\mu\omega_\nu^i - \Gamma_{\mu\nu}^\rho\omega_\rho^i + A_\mu{}^i{}_k\omega_\nu^k = 0 \tag{93}$$

which manifests the tetrade postulate resulting in solution (47). The latter represents $\Gamma_{\mu\nu}^\rho$ through the gauge field $A_\mu{}^i{}_k$. The inverse relation represents $A_\mu{}^i{}_k$ and its strength $F_{\mu\nu i}{}^k$ through $\Gamma_{\nu\lambda}^\rho$ and the Riemann tensor $R_{\mu\nu}{}^\gamma{}_\lambda$ (81)

$$A_\nu^{lm} = \omega_\rho^l \Gamma_{\nu\lambda}^\rho \omega^{\lambda m} + \omega_\lambda^l \partial_\nu \omega^{\lambda m}, \tag{94}$$

$$F_{\mu\nu}{}^i{}_k = \omega_\gamma^i R_{\mu\nu}{}^\gamma{}_\lambda \omega_k^\lambda. \tag{95}$$

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This means that the $SO(1,p)$ gauge field referred to the natural frame $\mathbf{e}_\mu(\xi)$ turns out to be the metric connection $\Gamma^\rho_{\nu\lambda}$. The substitutions of $\Gamma^\rho_{\nu\lambda}$ instead of the gauge field $A_{\nu ik}$, and the tensor field $l_{\mu\nu}{}^a = -\omega^i_\nu W_{\mu i}{}^a$ for $W_{\mu i}{}^a$ into the G-R-C constraints (75-77) transform them into the desired constraints [28]

$$R_{\mu\nu}{}^\gamma{}_\lambda = l_{[\mu}{}^{\gamma a} l_{\nu]\lambda a}, \tag{96}$$

$$H_{\mu\nu}{}^{ab} = l_{[\mu}{}^{\gamma a} l_{\nu]\gamma}{}^b, \tag{97}$$

$$\nabla^\perp_{[\mu} l_{\nu]\rho a} = 0, \tag{98}$$

where the general and $SO(D - p - 1)$ covariant derivative $\nabla^\perp_\mu$ is defined as

$$\nabla^\perp_\mu l_{\nu\rho}{}^a := \partial_\mu l_{\nu\rho}{}^a - \Gamma^\lambda_{\mu\nu} l_{\lambda\rho}{}^a - \Gamma^\lambda_{\mu\rho} l_{\nu\lambda}{}^a + B^{ab}_\mu l_{\nu\rho b}. \tag{99}$$

The commutator of these covariant derivatives yields the Bianchi identitities

$$[\nabla^\perp_\gamma, \nabla^\perp_\nu] l^{\mu\rho a} = R_{\gamma\nu}{}^\mu{}_\lambda l^{\lambda\rho a} + R_{\gamma\nu}{}^\rho{}_\lambda l^{\mu\lambda a} + H_{\gamma\nu}{}^a{}_b l^{\mu\rho b}. \tag{100}$$

Eqs. (96) generalize the *Gauss Theorema Egregium* for surfaces in 3-dim. Euclidean space to the case of (p+1)-dimensional world hypersurfaces embedded into D-dim. Minkowski space.

To identify Σ_{p+1} with the p-brane hypervolume we have to construct a gauge invariant $(p + 1)$-dim. action formulated in terms of the Cartan multiplets and to prove that its Euler-Lagrange EOM are compatible with constraints (96-98) and the EOM of a p-brane. The problem is solved in the next section according to the scheme described in Section 4.

7 Gauge invariant action for branes

The Dirac p-brane action [42] in D-dimensional Minkowski space equals the brane hypervolume including the determinant of the induced metric $g_{\mu\nu}$ (78)

$$S = T \int d^{p+1}\xi \sqrt{|det(\partial_\mu \mathbf{x} \partial_\nu \mathbf{x})|}. \tag{101}$$

This action yields the non-linear wave equation for the world vector $\mathbf{x}(\xi^\mu)$

$$\Box^{(p+1)} \mathbf{x} = 0, \tag{102}$$

where $\Box^{(p+1)}$ is the D'Alembert-Beltrami operator on the hypersurface Σ_{p+1}

$$\Box^{(p+1)} := \frac{1}{\sqrt{|g|}} \partial_\alpha \sqrt{|g|} g^{\alpha\beta} \partial_\beta. \tag{103}$$

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Eq. (102) means that the Dirac p-brane sweeps the minimal hypersurface Σ_{p+1} defined by the algebraic conditions

$$Spl^a \equiv g^{\mu\nu} l^a_{\mu\nu} = 0 \tag{104}$$

for the second fundamental form $l_{\mu\nu}{}^a$ defined by (92). The equivalence of these conditions to the wave Eq. (102) follows from Eqs. (85) as a result of the orthogonality between $\mathbf{n}^a$ and the vectors $\partial_\beta \mathbf{x}$ tangent to Σ_{p+1}: $\mathbf{n}^a \frac{\partial \mathbf{x}}{\partial \xi^\beta} = 0$ that makes the metric connection contribution vanishing.

In correspondence with the gauge theory reformulation of the broken Lagrangians discussed in Section 4 the minimality condition (104) plays the role similar to Eq. (51) representing the standard N-G EOM in the gauge approach. So, Eqs. (96-98), (104) are analogous to Eqs. (52), and yield a complete set of data to represent Dirac action (101) in terms of the Cartan multiplets. Such a representation is given by the below-discussed gauge-invariant action S_{Dir} (113) including a brane metric $g_{\mu\nu}$ which is considered as a given (background) field [28]. However, it does not mean that $g_{\mu\nu}$ is not a dynamical field, because its dynamics is already fixed by the Gauss condition (96) treated as the second order PDEs for $g_{\mu\nu}$ with a given $l^a_{\mu\nu}$. In its turn the dynamics of the N-G multiplet $l^a_{\mu\nu}$ is derived using the standard variational principle for the action (113). For the Nambu-Goto string (p=1) in 3-dimensional Minkowski space the metric condition (96) is the Lioville equation on string world-sheet showing that it is 2-dim. Einstein space

$$R_{\mu\nu} = \frac{R}{2} g_{\mu\nu}.$$

Keeping in mind such peculiarities in application of the variational principle for dynamical description of the metric and Cartan fields we consider derivation of (113) step by step. It will permit to avoid a possible missunderstanding of the presented results.

In the long wave approximation the leading order for a gauge invariant brane action has to be quadratic in the derivatives of the Cartan multiplets. To find such an action we start from a generalized Landau-Ginzburg action

$$S = \gamma \int d^{p+1}\xi \sqrt{|g|} \{ -\frac{1}{4} Sp(H_{\mu\nu} H^{\nu\mu}) + \frac{1}{2} \nabla_\mu^\perp l_{\nu\rho a} \nabla^{\perp\{\mu} l^{\nu\}\rho a}$$
$$- \nabla_\mu^\perp l^\mu_{\rho a} \nabla_\nu^\perp l^{\nu\rho a} + V \} \tag{105}$$

with a potential V encoding self-interaction of the N-G fields $l^a_{\mu\nu}$ and compatible with Eqs. (96-98). We prove that Eqs. (96-98), (104) permit to restore

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V in a unique fashion. The EOM for $B_\mu{}^{ab}$ and $l_{\mu\nu}{}^a$ following from (105) are

$$\nabla^\perp_\nu \mathcal{H}^{\nu\mu}_{ab} = \frac{1}{2} l_{\nu\rho[a} \nabla^{\perp[\mu} l^{\nu]\rho}{}_{b]}, \tag{106}$$

$$\frac{1}{2} \nabla^\perp_\mu \nabla^{\perp[\mu} l^{\{\nu]\rho\}a} = -[\nabla^{\perp\mu}, \nabla^{\perp\{\nu}] l_\mu{}^{\rho\}a} + \frac{\partial V}{\partial l_{\nu\rho a}}, \tag{107}$$

where $\mathcal{H}^{ab}_{\mu\nu} = H^{ab}_{\mu\nu} - l_{[\mu}{}^{\gamma a} l_{\nu]\gamma}{}^b$ is the shifted strenght $H^{ab}_{\mu\nu}$.

Then one can see that the Ricci and Codazzi eqs. (97-98) presented as

$$\mathcal{H}^{ab}_{\mu\nu} = 0, \qquad \nabla^\perp_{[\mu} l^a_{\nu]\rho} = 0, \tag{108}$$

form the solution of Eqs. (106-107) provided V satisfies the conditions

$$\frac{\partial V}{\partial l_{\nu\rho a}} = [\nabla^{\perp\mu}, \nabla^{\perp\{\nu}] l_\mu{}^{\rho\}a}. \tag{109}$$

The Bianchi identitities (100) permit to express the r.h.s. of (109) through $R_{\gamma\nu\rho\lambda}$, $H^{ab}_{\gamma\nu}$ and $l^a_{\mu\nu}$. Then conditions (109) could be transformed into PDEs for the scalar function $V(l)$ if $R_{\gamma\nu}{}^\rho{}_\lambda$ and $H_{\gamma\nu}{}^{ab}$ could be represented as some explicit functions of $l^a_{\mu\nu}$. Such a representation is given by the Gauss (96) and Ricci (97) conditions. This clarifies the dynamical role of these conditions as the selection rules *defining* V and the *metric* in S (105). The substitution of Eqs. (96-97) in (109) transforms the latter into the PDEs for the potential $V(l)$ *fixing* it to be the invariant *quartic* polynomial in $l_{\mu\nu}{}^a$

$$\frac{1}{2} \frac{\partial V}{\partial l_{\nu\rho a}} = \frac{1}{2} (l^{\{a} l^{b\}})^{\rho\nu} Spl_b + (2l_b l^a l^b - l^a l_b l^b - l_b l^b l^a)^{\rho\nu} - l_b^{\rho\nu} Sp(l^b l^a) \tag{110}$$

where $Sp(l^a l^b) = g^{\mu\nu} l^a_{\mu\rho} l^{b\rho}_\nu$. The solution corresponding to Dirac branes is

$$V_{Dir} = -\frac{1}{2} Sp(l_a l_b) Sp(l^a l^b) + Sp(l_a l_b l^a l^b) - Sp(l_a l^a l_b l^b) + c, \quad Spl^a = 0 \tag{111}$$

with the integration constant c. The minimality condition in (111) is invariant under all symmetries of S (105), and is interpreted as the inverse Higgs phenomenon condition additional to $\omega_a = 0$. Then EOM (107) reduce to

$$\frac{1}{2} \nabla^\perp_\mu \nabla^{\perp[\mu} l^{\{\nu]\rho\}a} \equiv \nabla^\perp_\mu (\nabla^{\perp[\mu} l^{\nu]\rho a} + \frac{1}{2} \nabla^{\perp[\nu} l^{\rho]\mu a}) = 0 \rightarrow \nabla^\perp_\rho \nabla^\perp_\mu l^{\mu\rho a} = 0 \tag{112}$$

where we used the identity $\nabla^{\perp[\mu} l_a^{[\nu]\rho]} = -\nabla^{\perp[\nu} l_a^{\rho]\mu}$.

As a result, we obtain the desired invariant action for the Dirac p-brane

$$S_{Dir} = \gamma \int d^{p+1}\xi \sqrt{|g|}\; \mathcal{L},$$

$$\mathcal{L} = -\frac{1}{4}Sp(H_{\mu\nu}H^{\nu\mu}) + \frac{1}{2}\nabla_\mu^\perp l_{\nu\rho a}\nabla^{\perp\{\mu}l^{\nu\}\rho a} - \nabla_\mu^\perp l_{\rho a}^\mu \nabla_\nu^\perp l^{\nu\rho a}$$

$$-\frac{1}{2}Sp(l_a l_b)Sp(l^a l^b) + Sp(l_a l_b l^a l^b) - Sp(l_a l^a l_b l^b) + c \qquad (113)$$

desribing the interacting traceless tensor $l_{\mu\nu}^a$ and vector B_μ^{ab} multiplets in the background metric $g_{\mu\nu}$. Thereat, the dynamical equations for $g_{\mu\nu}$ are encoded by the Gauss conditions (96) automatically built in S_{Dir}.

The Euler-Lagrange EOM for S_{Dir} (113) can be written in the form

$$\nabla_\nu^\perp \mathcal{H}_{ab}^{\nu\mu} = \frac{1}{2}l_{\nu\rho[a}\nabla^{\perp[\mu}l^{\nu]\rho}{}_{b]}, \qquad (114)$$

$$\nabla_\mu^\perp \nabla^{\perp[\mu}l^{\nu]\rho a} + \frac{1}{2}\nabla^{\perp[\nu}\nabla_\mu^\perp l^{\rho]\mu a} = 0, \qquad (115)$$

of the *second order* PDEs after using Eqs. (104), (112) and the identities

$$[\nabla^{\perp\mu}, \nabla^{\perp[\nu}]l_\mu^{\rho]a} = ([l^b, l^a])^{\nu\rho}Spl_b \qquad (116)$$

which follow from the Bianchi identities (100).

Now we prove that Eqs. (108) yield the *general solution* of (114-115). Taking into account that (108) are PDEs of the *first order* we consider them to be the *Cauchy initial data* for PDEs (114-115) chosen at the time $\tau = 0$

$$\mathcal{H}_{ab}^{\nu\mu}(0, \sigma^r) = 0, \quad \nabla^{\perp[\mu}l_a^{\nu]\rho}(0, \sigma^r) = 0, \quad \nabla_\mu^\perp l_a^{\mu\rho}(0, \sigma^r) = 0 \qquad (117)$$

and show that the R-C Eqs. (108) are always satisfied in view of EOM (114-115). Using the power series expansion of $\mathcal{H}_{ab}^{\nu\mu}$ and $\nabla^{\perp[\mu}l^{\nu]\rho a}$

$$\mathcal{H}_{ab}^{\tau r}(\delta\tau, \sigma^r) = \mathcal{H}_{ab}^{\tau r}|_{\tau=0} + \partial_\tau \mathcal{H}_{ab}^{\tau r}|_{\tau=0}\delta\tau + ... = \nabla_\tau^\perp \mathcal{H}_{ab}^{\tau r}|_{\tau=0}\delta\tau + ..., \quad (118)$$

$$\nabla^{\perp[\tau}l^{\nu]\rho a}(\delta\tau, \sigma^r) = \partial_\tau \nabla^{\perp[\tau}l^{\nu]\rho a}|_{\tau=0}\delta\tau + ... = \nabla_\tau^\perp \nabla^{\perp[\tau}l^{\nu]\rho a}|_{\tau=0}\delta\tau + ...$$

and Eqs. (114-115), (117) we obtain

$$\mathcal{H}_{ab}^{\tau r}(\delta\tau, \sigma^r) = -\nabla_{r'}^\perp \mathcal{H}_{ab}^{r'r}|_{\tau=0}\delta\tau + ..., \qquad (119)$$

$$\nabla^{\perp[\tau}l^{\nu]\rho a}(\delta\tau, \sigma^r) = -(\nabla_{r'}^\perp \nabla^{\perp[r'}l^{\nu]\rho a} + \frac{1}{2}\nabla^{\perp[\nu}\nabla_\mu^\perp l^{\rho]\mu a})|_{\tau=0}\delta\tau + ...$$

Then observing that the space covariant derivatives of (117) are equal to zero

$$\nabla^\perp_{r'}\mathcal{H}^{\nu\mu}_{ab}|_{\tau=0} = \nabla^\perp_{r'}\nabla^{\perp[\mu}l^{\nu]\rho a}|_{\tau=0} = \nabla^\perp_{r'}\nabla^\perp_\mu l^{\mu\rho}_a|_{\tau=0} = 0, \quad (r' = 1, 2, ..., p)$$

derive conservation of the part of desired Ricchi-Codazzi conditions

$$\mathcal{H}^{\tau r}_{ab}(\tau, \sigma^r) = 0, \quad \nabla^{\perp[\tau}l^{r]\rho}_a(\tau, \sigma^r) = 0, \quad \nabla^\perp_\mu l^{\mu\rho}_a(\tau, \sigma^r) = 0. \quad (120)$$

Vanishing of the magnetic components $\mathcal{H}^{sr}_{ab}$ follows from their definition and the first pair of the Maxwell equations for $H^{\mu\nu}_{ab}$ resulting in the equations

$$\sum_{cycle(\mu\nu\rho)} (\nabla^\perp_\mu \mathcal{H}^{ab}_{\nu\rho} + \nabla^\perp_{[\mu}l^{[a}_{\nu]\lambda}l^{b]\lambda}_\rho) = 0, \quad (121)$$

where $\sum_{cycle(\mu\nu\rho)}$ denotes the sum in the cyclic permutations of the μ, ν, ρ indices. Eqs. (121) are easily derived using the identities

$$\sum_{cycle(\mu\nu\rho)} \nabla^\perp_\mu (l^a_{\lambda[\nu}l^{\lambda b}_{\rho]}) = \sum_{cycle(\mu\nu\rho)} \nabla^\perp_{[\mu}l^{[a}_{\nu]\lambda}l^{b]\lambda}_\rho. \quad (122)$$

By combining equations (121), (120) with the Cauchy data (117) we obtain $\mathcal{H}^{sr}_{ab}(\delta\tau, \sigma^r) = 0$. The substitution of all the above found solutions to (112) and (114) permits to show that $\nabla^{\perp[r}l^{s]\tau}_a(\delta\tau, \sigma^r) = \nabla^{\perp[r}l^{s]q}_a(\delta\tau, \sigma^r) = 0$. The latter proves conservation of the complete set of the Codazzi conditions $\nabla^{\perp[\mu}l^{\nu]\rho}_a = 0$. Thus, the R-C eqs. (108) are actually conserved in time

$$\mathcal{H}^{ab}_{\mu\nu}(\delta\tau, \sigma^r) = \mathcal{H}^{ab}_{\mu\nu}(0, \sigma^r), \qquad \nabla^\perp_{[\mu}l^a_{\nu]\rho}(\delta\tau, \sigma^r) = \nabla^\perp_{[\mu}l^a_{\nu]\rho}(0, \sigma^r) \quad (123)$$

together with their consequences $\nabla^\perp_\mu l^{\mu\rho}_a(\tau, \sigma^r) = 0$ following from the minimality conditions (104). In correspondence with the Cauchy-Kowalevskaya theorem of local existence and uniqueness, we see that Eqs. (108) define the covariant solution of EOM (114-115) modulo the gauge and diffeomorphism symmetries of S_{Dir}. Then these EOM become equivalent to the identities

$$\nabla^{\perp\mu}\mathcal{H}^{ab}_{\mu\nu} = 0, \qquad \nabla^{\perp\mu}\nabla^\perp_{[\mu}l^a_{\nu]\rho} = 0 \quad (124)$$

produced by the covariant differentiation of the Ricci-Codazzi Eqs. (108), and can be equivalently written in the form of the generalized Maxwell-Y-M and Newton equations in the gravitational field defined by Eqs. (96)

$$\nabla^\perp_\nu H^{\nu\mu}_{ab} = j^\mu_{ab}, \qquad j^\mu_{ab} = Sp(l_{[a}\nabla^{\perp\mu}l_{b]}), \quad \nabla^\perp_\mu j^\mu_{ab} = 0, \quad (125)$$

$$\nabla^\perp_\mu \nabla^{\perp\mu}l^{\nu\rho a} = \frac{1}{2}\frac{\partial V_{Dir}}{\partial l_{\nu\rho a}} \equiv (2l_b l^a l^b - l^a l_b l^b - l_b l^b l^a)^{\nu\rho} - l^{\nu\rho}_b Sp(l^b l^a). \quad (126)$$

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We conclude that S_{Dir} (113) with the chosen potential V_{Dir} (111) reformulates the Dirac p-brane dynamics it terms of the Cartan multiplets. The particular solution $V = const, \ l_{\mu\nu}^a = 0$ describes flat branes with $g_{\mu\nu} = \eta_{\mu\nu}$.

In the original brane action S (101) the N-G translational modes are condenced in the volume of the coset $ISO(1, D-1)/SO(1, D-1)$ expressed through the derivatives of $\mathbf{x}$. This action is the leading term in the long-wave description of the brane dynamics. In the action S_{Dir} (113) the translational modes form the background metric $g_{\mu\nu}$ treated as an independent field. The broken translational and rotational modes condenced in $l_{\mu\nu}^a$ associate with the extrinsic curvature of Σ_{p+1}. The Gauss conditions (96) connect the rotational and translational N-G modes and defines dynamics of the metric field $g_{\mu\nu}$. S_{Dir} also contains the cosmological term that points to spontaneous breakdown of the global Poincare symmetry of the Minkowski space.

8   Summary

The gauge theory approach to branes was interpreted in the language used for the system with spontaneously broken internal symmetries. However, in contrast to the standard description of the Nambu-Goldstone fields as co-ordnates of a coset G/H, we considered their purely geometric description without any explicit parametrization. It was based on the use of moving frames, the Cartan forms and the *right gauge* symmetries H_R. These symmetries remove auxiliary N-G modes corresponding to the generators of the vacuum subgroups H of the completely broken global symmetry G. This shows equivalency of the N-G fields to the Cartan multiplets formed by the constrained vector and Yang-Mills multiplets of H_R. Then we extended this approach to p-branes embedded into Minkowski space $\mathbf{R}^{1,D-1}$ invariant under the global Poincare symmetry $ISO(1, D-1)$. The latter was treated as the symmetry spontaneously broken by the presence of a minimal brane hypersurface Σ_{p+1}. We treated the orthonormal vectors of the Cartan moving frame $\mathbf{n}_A(\xi)$ attached to Σ_{p+1} together with its world vector $\mathbf{x}(\xi)$ as the order parameters fixing a macroscopic vacuum manifold of p-brane represented by Σ_{p+1}. The symmetry group of the vacuum manifold was fixed by the condition of vanishing for the translational Cartan forms ω_a orthogonal to Σ_{p+1}. This resulted in emergence of constrained Cartan multiplets of the unbroken subgroup $SO(D-p-1) \in SO(1, D-1)$, their gauge invariant interaction potential V_{Dir} (111) and the action S_{Dir} (113). The multiplet

constraints were treated as the conserved Cauchy data for the corresponding Euler-Lagrange EOM. Thus, S_{Dir} was shown to give an alternative description of the fundamental p-branes by the Yang-Mills (125) and Newton (126) equations. Thereat, the evolution of p-brane metric was encoded by the Gauss conditions (96) treated as the second order PDEs. For co-dimension 1 $H^{ab}_{\mu\nu} = j^{ab}_{\mu} \equiv 0$ and Eqs. (125-126) reduce to the (p+1)-dim. equation

$$\Box l_{\nu\rho} = l_{\nu\rho} Sp(l^2), \qquad Spl = 0, \qquad (127)$$

where $\Box \equiv \nabla_{\mu}\nabla^{\mu}$ is the D'Alembert-Beltrami operator for a tensor field on Σ_{p+1}. Eq. (127) coincides with the Laplace-Beltrami equation for the second fundamental form of minimal hypersurfaces in Euclidean space ([44],[45]), but with the $\Box$ operator substituted for the Laplace-Beltrami one. [5]

The gauge approach can be generalized to the case of D-branes or M-theory branes. This will modify the potential V in the action (105). It is also interesting to quantize S_{Dir} using the well-known BRST-BFV method. Quantization may fix the cosmological constant value and shed new light on the problems connected with ghosts, anomalies and critical dimensions.

Acknowledgments

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